

5.1 Differentiation

Question Paper

Course	CIEAS Maths
Section	5. Differentiation
Topic	5.1 Differentiation
Difficulty	Very Hard

Time allowed: 70
Score: /55
Percentage: /100

Question 1a

For each of the following, find $\frac{dy}{dx}$ in terms of x :

(a) $y = -\frac{5}{4}x^3 + \frac{3}{5}x^2 - x\sqrt{2} + \pi$

[2 marks]

Question 1b

(b) $y = \frac{3}{2}x^{\frac{4}{5}} - \frac{10}{3}x^{-\frac{4}{5}}$

[2 marks]

Question 2

Given that $y = \left(\frac{1}{x} - \frac{1}{x\sqrt{x}}\right)^2$, $x > 0$, find $\frac{dy}{dx}$.

[4 marks]

Question 3a

For each of the following, find $\frac{dy}{dx}$ in terms of x :

(a) $y = \frac{2x^3 - 5x^2 - 3x}{2x + 1}$

[3 marks]

Question 3b

(b) $y = \left(\sqrt{x} + 3 - \frac{4}{\sqrt{x}}\right)^2$

[4 marks]

Question 4a

For each of the following, use the chain rule to find $\frac{dy}{dx}$ in terms of x :

(a) $y = \left(\frac{1}{\sqrt{x}} + 3x^2\right)^3$

[4 marks]

Question 4b

(b) $y = \sqrt{\frac{1}{\sqrt{x} + \frac{1}{\sqrt{x}}}}$

[4 marks]

Question 5

A curve has the equation $y = x\sqrt{x} + \frac{48}{\sqrt{x}}$, $x > 0$. Find the coordinates of the point on the curve where the gradient is 0.

[5 marks]

Question 6

The function f is defined by $f(x) = 2x^3 + px^2 + 3x - 16$. Determine the range of values for p for which the equation $f'(x) = 0$ has at least one real solution.

[5 marks]

Question 7a

Given that $y = \left(\frac{1}{x} + x\right)^4$, $x \neq 0$

(a) use the chain rule to find $\frac{dy}{dx}$

[3 marks]

Question 7b

(b) find the coordinates of any stationary points and determine their nature

[4 marks]

Question 7c

(c) sketch the curve.

[3 marks]

Question 8

The function f is defined by $f(x) = x^n - x$, $n \in \mathbb{N}, n \geq 2$. Determine the relationship between the value of n and the number of real solutions to the equation $f'(x) = 0$.

[4 marks]

Question 9

A curve is described by the equation $\frac{\sqrt{y}}{-1 + \sqrt{x}} = \frac{1}{x}$, $x > 1$. Find $\frac{dy}{dx}$.

[3 marks]

Question 10

The curve with equation $y = ax^2 + bx + c$ passes through the point $(-1, 4)$. At the point $(2, 7)$ the gradient of the curve is 7. Find the values of a , b and c .

[5 marks]